Static/flowing transition and erosion processes in granular flows: laboratory experiments and numerical modelling

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Rheological behavior of granular materials

Static/flowing transition and in particular erosion processes?

Laboratory experiments ↔ Physical and numerical modelling
Static/flowing transition in granular flows on erodible beds


Friction angles:
repose \( \delta \approx 23^\circ \pm 0.5^\circ \), avalanche \( \delta \approx 25^\circ \pm 0.5^\circ \)

Control parameters:
- slope angle: \( 0^\circ < \theta < \delta \)
- volume \( V = h_0 r_0 W \)
  - aspect ratio \( a = h_0 / r_0 \)
  - erodible bed thickness \( h_i \)
  - column shape
  - degree of bed compaction
  - channel width

What is the impact on flow dynamics, deposition and erosion processes?

Mangeney et al. 2010, Farin et al. 2014
Granular column collapse over inclined channels

Rigid bed

Erodible bed

Mangeney et al. 2010
Granular collapse over an erodible bed

Runout distance can \( \sim \) by up to 40%.

\(~ \theta_c : \text{Critical slope angle over which the presence of an erodible bed affects the flow}\)

Farin, Mangeney, Roche 2014
Erosion depth is lower and erosion duration is greater during the deceleration and slow propagation phases => runout distance

\[ \theta = 23^\circ \]

Within the erodible bed

Farin, Mangeney, Roche 2014
Time change of the static/flowing interface up to the stopping of the whole mass

Very strong constrain for mechanical models!!
2D granular flow modelling

Non-dimensional form

• Momentum equation: 
  \[ \rho \left( S_t \frac{\partial u}{\partial t} + (u \cdot \nabla)u \right) - \text{div} \ \sigma' + \nabla p = \frac{1}{Fr^2} \rho f \]

• Incompressibility: 
  \[ \text{div} \ u = 0 \]

• Boundary conditions:
  - At the base and wall: 
    \[ u \cdot n = 0, \ \left\{ \begin{array}{l} \sigma_T = \mu_b \sigma_n \frac{v}{|v|} \quad \text{if } |v| \neq 0, \\ |\sigma_T| \leq \mu_b \sigma_n \quad \text{if } |v| = 0, \end{array} \right. \]

  - At the free surface: \( \mathcal{D}(t) \) characteristic function of the domain
    \[ \sigma n = 0 \quad \text{on} \quad \Gamma_s(t), \quad S_t \frac{\partial 1_{\mathcal{D}(t)}}{\partial t} + u \cdot \nabla 1_{\mathcal{D}(t)} = 0, \]

• Initial conditions: 
  \[ u|_{t=0} = 0 \ , \ \mathcal{D}(0) = \mathcal{D}_0. \]
Constitutive relation for granular material

\[ \sigma' = 2\eta(|D|, p)D + \kappa(p)\frac{D}{|D|} \quad \text{if } |D| \neq 0, \]
\[ |\sigma'| \leq \kappa(p) \quad \text{if } |D| = 0. \]

The plasticity (flow/no flow) criterion: \( \kappa(p) = \mu_s p \)

Strain rate tensor: \( \dot{D}(u) = \frac{1}{2} (\nabla u + \nabla^T u) \), pressure: \( p = -\text{trace}(\sigma)/3 \)

- Constant viscosity: \( \eta(|D|, p) = \text{Cte} \)

- \( \mu(I) \leftrightarrow 2\eta(|D|, p) = \frac{k(\mu_2 - \mu_s)p}{k|D| + I_0 \sqrt{p}}, \quad k = d\sqrt{\rho_s} \)

\[ \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{1 + I_0/I} \quad \text{with } I = \frac{2\|D\|d}{\sqrt{p/\rho_s}} \]

Ionescu, Mangeney, Bouchut, Roche 2015
Parameters deduced from the experiments

Grain diameter: \( d = 0.7 \pm 0.1 \text{mm} \)

Density: \( \rho_s = 2500 \ \text{kg} \ \text{m}^{-3} \), \( \nu = 0.62 \quad \Rightarrow \quad \rho = 1550 \ \text{kg} \ \text{m}^{-3} \)

Repose angle: \( \theta_r = 23.5^\circ \pm 0.5^\circ \) \( (\mu_r = 0.43 \pm 0.01) \)

Avalanche angle: \( \theta_a = 25.5^\circ \pm 0.5^\circ \) \( (\mu_a = 0.48 \pm 0.01) \)

Wall friction: \( \mu_w = \tan(10.5^\circ) = 0.18 \) \hspace{1cm} Poulquen and Forterre 2002

Additional friction due to the wall: \( +\mu_w h/W \) \hspace{1cm} Jop et al. 2005

\[
\begin{cases}
\text{Drucker-Prager:} & \mu = \mu_s = \tan(25.5^\circ), \eta \\
\text{Jop et al. 2005:} & \mu_s = \tan(25.5^\circ), \mu_2 = \tan(36^\circ), I_0 = 0.279
\end{cases}
\]

Friction at the base: \( \mu_b = \mu = \tan(25.5^\circ) \)
Simulation with the variable viscosity ($\mu (I)$)

Granular collapse over an horizontal plane: $\theta = 0^\circ$

Parameters deduced from experiments:

$$\mu_s = \tan(25.5^\circ), \quad \mu_2 = \tan(36^\circ), \quad I_0 = 0.279, \quad \mu_b = \mu_s = \tan(25.5^\circ)$$

Well reproduces the dynamics

The gate has to be taken into account!
Viscosity $\eta$ in the $\mu (I)$ rheology

$2\eta(|D|, p) = \frac{k(\mu_2 - \mu_8)p}{k|D| + I_0 \sqrt{p}}$,

Very similar results with $\eta = 1$ Pa.s
Insight into the flow dynamics

Flow heterogeneities: strain rate localization and pressure fluctuations

Good results but high computational cost!
Multilayer Shallow Model

For application to natural flows, equations have to be simplified to lower the computational cost!

Thin-layer (i.e. shallow) approximation  \( a = h/L \ll 1 \)

\[ p = g \cos \theta (h - Z) \] pression hydrostatique

Fernandez-Nieto, Garres, Mangeney, Narbona-Reina, 2015
Monolayer (Saint-Venant) versus Multilayer models

\[ \mu_s = \tan(25.5^\circ) \approx 0.48 \]

\[ \theta = 22^\circ \]

\[ \mu(I) \]
Erosion effects on avalanche runout

\[ \mu_s = \tan(25.5^\circ) \approx 0.48 \]

How to get quantitative agreement?

\[ \mu(I) \] in the Multilayer Shallow Model reproduces qualitatively the increase of runout due to entrainment on sloping erodible beds.
Modelling of the static/flowing transition

Reduce computational cost!

Two thin-layer depth-averaged model: a flowing layer above a static layer

Equation for the static/flowing transition $\dot{b}(t)$?

UP TO NOW
Arbitrary closure relations and non-consistent energy

*Fernandez-Nieto, 2008, Iverson and Ouyang, 2015*

Go back to non-depth-averaged models

Viscoplastic models CONTAIN the static/flowing transition without having to prescribe arbitrary exchange rates, velocity profiles, etc…

*Bouchut, Ionescu, Mangeney, 2015*
Simplified 1D shallow viscoplastic model

\[ \partial_t U(t, Z) + S(t, Z) - \partial_Z \left( \nu \partial_Z U(t, Z) \right) = 0 \]
\[ S = g(-\sin \theta + \partial_X (h \cos \theta)) - \mu_s \partial_Z p \]

Thin-layer approximation

\[ p = g \cos \theta (h - Z) \Rightarrow S = g(-\sin \theta + \mu_s \cos \theta) \]

Boundary conditions

\[
\begin{align*}
U &= 0 \quad \text{at} \quad Z = b(t), \\
\nu \partial_Z U &= 0 \quad \text{at} \quad Z = b(t), \\
\nu \partial_Z U &= 0 \quad \text{at} \quad Z = h.
\end{align*}
\]

Initial condition

\[ U(0, Z) = U^0(Z) \]

Static/flowing interface

- If \( \nu = 0 \) and \( \partial_Z U(t, b(t)) \neq 0 \), then
  \[ \dot{b}(t) = \frac{S(t, b(t))}{\partial_Z U(t, b(t))} \]
  \[ U(t, Z) = \max \left( U^0(Z) - S t, 0 \right) \]

- If \( \nu > 0 \) and \( S(t, b(t)) \neq 0 \), then
  \[ \dot{b}(t) = \left( \frac{\partial_Z S(t, b(t)) - \nu \partial^3_Z U(t, b(t))}{S(t, b(t))} \right) \nu \]

Lusso, Bouchut, Ern, Mangeney, 2015a
Simplified 1D shallow viscoplastic model

Initial linear velocity profile

Initial exponential velocity profile

Experimental results

No viscosity

No penetration within the initially static bed
Simplified 1D shallow viscoplastic model

Experimental results

with viscosity

the initially static bed is put into motion
Conclusion

- Drucker-Prager with a constant viscosity and $\mu(I)$ rheology reproduce quantitatively granular collapse experiments
  
  Insight into flow dynamics

- Only the $\mu(I)$ rheology reproduces qualitatively the increase in runout when the thickness of the erodible bed increases

- Still to reproduce quantitatively erosion effects

A big challenge for application to natural landslides

- Derive from Drucker-Prager equations two layers Saint-Venant equations: a flowing layer over a static layer
Effect of the gate on the collapse dynamics

$\theta = 0^\circ$

experiments

$\text{gate}$

$\text{no gate}$

$t = 0.06s$

$t = 0.18s$

$t = 0.3s$

$t = 1.02s$

same deposit!
Very similar results with the variable and constant viscosity $\eta = 1 \text{ Pa.s}$
Horizontal velocity field and profiles

$\theta = 0^\circ$

Good results but high computational cost!
- iterative decomposition-coordination formulation
- coupled with the augmented Lagrangian method
- ALE (Arbitrary Lagrangian-Eulerian) description to compute the evolution of the fluid domain $D(t)$.

Mesh refinement around the free surface

*Ionescu, Mangeney, Bouchut, Roche, 2014*
\[ I = \frac{2 \| D \| d}{\sqrt{\rho / \rho_s}} \]

\[ \mu(I) = \mu_s + \frac{\mu_2 - \mu_s}{1 + I_0/I} \]
Granular flow dynamics

acceleration/deceleration

Rigid bed

$a = 0.7 - V = 12600 \text{ cm}^3$

- $\theta = 0^\circ$
- $\theta = 10^\circ$
- $\theta = 16^\circ$
- $\theta = 22^\circ$

=> 2 flow regimes

slow propagation phase $\approx$ steady uniform flow Poulquen 1999

Increase of the duration of the slow propagation phase - when slope angle

- over an erodible bed

\[ \theta = 22^\circ - a = 0.7 - V = 12600 \text{ cm}^3 \]

\[ \theta = 22^\circ \text{ rigid bed} \]

\[ \theta = 22^\circ \text{ erodible bed, } h_i = 4 \text{ d} \]
Kelvin-Helmholtz instabilities?

5 cm < \lambda < 10 cm

Waves behind the front

Kelvin-Helmholtz instabilities?
Thin Layer Approximation (Saint-Venant)

- Flow on complex natural topography

\[ \downarrow \quad \text{small Aspect ratio} \]

high computational cost \[ \Rightarrow \]

\[ a = \frac{H}{L} \ll 1 \]

- Depth-averaged thin layer model model

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial X} = \gamma_X g - K \frac{\partial}{\partial X} (g \gamma_Z h) - \mu \left( g \gamma_Z + \frac{u^2}{R_x} \right) \frac{|u|}{u} \]

\[ \gamma_X = \sin \theta, \quad \gamma_Z = \cos \theta \]

Savage and Hutter, 1989
Large variety of natural flows

Empirical simulation using thin-layer depth-averaged models with friction

\[
\mu(U) = \frac{\mu_o - \mu_w}{(1 + \|U\|/U_w)} + \mu_w, \quad \mu_o = 0.84, \quad \mu_w = 0.11, \quad U_w = 4 \text{ m.s}^{-1}
\]

Lucas, Mangeney, Ampuero, Nature Communications, 2014
Empirical friction weakening law

\[ \mu(U) = \frac{\mu_o - \mu_w}{(1 + \|U\|/U_w)} + \mu_w \]

\[ \mu_o = 0.84, \mu_w = 0.11, U_w = 4 \text{ m.s}^{-1} \]

Lucas, Mangeney, Ampuero, Nature Communications, 2014
Reproduce small to large landslides

Exemple : Fei Tsui (Hong-Kong)

With the same parameters

Improve deposit structure

Friction coefficient $0.1 < \mu < 0.8$

Constrains through seismic wave analysis

Time $t=1$ s

Time $t=3$ s

Time $t=9$ s

Thickness [m] Velocity [m/s] $\mu(U)$
Simulation of the generated seismic waves

The scenario with glacier better reproduces the vertical waveform.